Verification Method of Ratio Error of Direct Current Comparator Bridge in Standard Resistor Measurement

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Abstract

This work presents a verification method for determining the ratio error of a Direct Current Comparator (DCC) bridge, model 6010C, in primary resistance metrology. The proposed R-group method employs three CCC-calibrated standard resistors with proven long-term stability to evaluate the 1:1 resistance ratio error across resistance ranges of 1 Ω, 100 Ω, and 10 kΩ. The measured ratio errors were − 0.082 μΩ/Ω, − 0.032 μΩ/Ω, and 0.061 μΩ/Ω, respectively, with combined standard uncertainties (k = 1) in the range of 0.022 μΩ/Ω to 0.025 μΩ/Ω, which are lower than those obtained using the traditional Exchanging method. All calculated E_n ratios were below one, confirming the consistency and reliability of the results. The findings indicate that the R-group method provides a precise and efficient alternative for DCC bridge verification and a guideline for calibration, particularly in testing/calibration laboratories without access to a cryogenic current comparator.

Keywords: Standard Resistor, Direct Current Comparator Bridge (DCC bridge), Calibration, Verification

1. Introduction

The Direct Current Comparator (DCC) bridge [1] is an instrument widely used in primary resistance metrology for measuring the resistance ratio between two 4-terminal standard resistors, denoted as R_1 and R_2, as shown in Figure 1. This system provides a base relative accuracy of 1×10^{-7} within the intermediate resistance range from 1 Ω to 10 kΩ.

In the operation of a DCC bridge, two current sources are utilized: the primary current (I_1) and the secondary current (I_2). These currents flow through the primary winding (N_1) and secondary winding (N_2), respectively. As current passes through each winding, magnetomotive forces, N_1I_1 and N_2I_2, are generated. The DCC bridge system maintains balance by automatically adjusting both the number of turns in the secondary winding (N_2) and the magnitude of the secondary current (I_2), as described in reference [1]. Equilibrium is achieved when the resulting magnetic flux in the core is zero, which is typically indicated by a null reading on the galvanometer (G) or precision reference voltmeter. At this point, the voltage drop across two standard resistors can be measured accurately. The relationships among these parameters are described in equations (1) and (2).

$$I_1R_1 = I_2R_2 \tag{1}$$

$$I_1N_1 = I_2N_2 \tag{2}$$

In the measurement process, both the forward current and the backward current must be measured to reduce the impact of thermal electromotive force (EMF) caused by temperature differences at the metal junctions [2]. The Cryogenic Current Comparator (CCC) [3], which has higher accuracy than DCC, is used as a reference standard to calibrate DCC by comparing the ratio measurement of the same pair of resistors. The calibration results indicate that the uncertainty of the DCC resistance ratios is less than 10^{-8} Ω/Ω, [4].

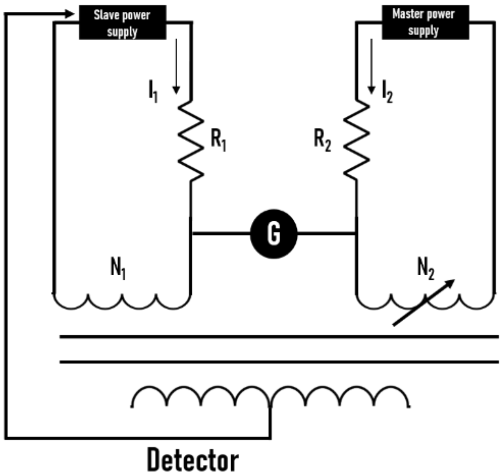

Fig. 1 The basic electrical circuit of a DCC bridge using a galvanometer for voltage measurement.

In precision resistance ratio measurements using a DCC bridge, the bridge ratio errors must be carefully considered. Such errors can arise from factors including winding deviations within the bridge, residual magnetic flux, hysteresis effects, and the sensitivity and stability limits of nanovolt-level measurements [4–5]. Consequently, calibration of the DCC bridge is essential to quantify these ratio errors and to assess the uncertainty of the measured resistance ratio. Previous studies have commonly employed the two–reference–resistor Exchanging method for DCC bridge verification [2]. The aim of this work is to validate the R-group method for DCC bridge calibration in the absence of a CCC bridge. This work presents the R-group method for determining the ratio error of DCC bridges using three standard resistors calibrated with a CCC bridge. The method is

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applied to evaluate the ratio error of a commercial DCC bridge (model 6010C) at 1:1 ratios of 1 Ω, 100 Ω, and 10 kΩ, with measurement configurations set according to the manufacturer's specifications. Validation is performed by comparison with the conventional Exchanging method, with the E_n ratio assessed for each resistance range. A comprehensive uncertainty evaluation of the DCC bridge using the R-group method is conducted to ensure that the method is efficient and reliable.

2. Methodology

2.1 R-group Method

The R-group method is a method developed by measuring the resistance values in a group of standard resistors. Based on the relationship between the resistance values in the group, the DCC ratio error can be checked and determined. The R-group method uses at least three standard resistors to measure a 1:1 ratio of the DCC bridge. The ratio error is determined by calculating the difference between the measured value and the certified values of the standard resistors, as shown in equation (3):

$$\text{Error value} = \text{Measured value} - \text{True value} \tag{3}$$

If the measured value does not equal the true value, the resistance ratio of the DCC bridge can be assumed as follows in equations (4) to (6).

$$\frac{R_1}{R_2} = X_1 - \Delta Y_0 \tag{4}$$

$$\frac{R_2}{R_3} = X_2 - \Delta Y_0 \tag{5}$$

$$\frac{R_3}{R_1} = X_3 - \Delta Y_0 \tag{6}$$

Equations (4), (5), and (6) are multiplied and equal to one due to all multiplied resistance ratios, as shown in equation (7).

$$1 = (X_1 - \Delta Y_0)(X_2 - \Delta Y_0)(X_3 - \Delta Y_0) \tag{7}$$

The ratio error of the DCC bridge can be obtain as in equation (8)

$$\Delta Y_0 = \frac{-1 + X_1 X_2 X_3}{X_1 X_2 + X_1 X_3 + X_2 X_3} \tag{8}$$

Where ΔY_0 is the bridge ratio error, and X_1, X_2, X_3 are the resistance ratios of R_1/R_2, R_2/R_3, and R_3/R_1, respectively, measured by the DCC bridge.

2.2 Instrument and Measurement

The DCC bridge used in this study is the MI 6010C (Measurements International Model 6010C) [6]. For the 1:1 ratio measurement by the R-group method, three standard resistors are used. They are arranged in three pairs, each pair containing two resistors. The DCC bridge is set up according to the manufacturer's manual, with further details provided in Table 1. The two standard resistors are connected to the DCC bridge, as shown in Figure 2.

In this experiment, 1 Ω, 100 Ω, and 10 kΩ standard resistors were used as the primary reference standards. The R_1, R_2, and R_3 standard resistors were calibrated by the Bureau International des Poids et Mesures (BIPM) using a Cryogenic Current Comparator (CCC) bridge, which provides high accuracy, stability, and reliability, with a measurement uncertainty of 1×10^{-9} Ω/Ω. Each standard resistor has a measurement history of more than ten years, allowing the prediction of short-term resistance changes in the part-per-billion (ppb) per day range. For the standard resistors used in this work, three 1 Ω resistors (in oil) were employed: two Leeds & Northrup 4210 and one Leeds & Northrup 4210B. For the 100 Ω standard, three resistors were used: two in air (Tegam SR102 and IET SR102) and one Tinsley 5685A (in oil). For the 10 kΩ standards, two Tegam SR104 (in air) and one Leeds & Northrup 4214 (in oil) were used. For the 10 kΩ standards, two Tegam SR104 (in air) and one Leeds & Northrup 4214 (in oil) were used. The resistors immersed in oil baths were maintained at a controlled temperature of (23.0 ± 0.5) °C. The air-stored resistors were kept in a laboratory environment controlled at (23.0 ± 2.0) °C and relative humidity of (50 ± 15) %RH. In this work, measurements were performed by alternately rotating the connection of the standard resistors in both clockwise and counterclockwise sequences.

In each measurement cycle, each resistance ratio was collected 35 times repeatedly, with the first 10 values discarded, and the remaining 25 values were then calculated to obtain the mean and standard deviation. This approach allows verification of the measured ratio error and provides an assessment of the repeatability of the measurement results.

Table 1 Measurement configurations of DCC bridge model 6010C for 1:1 resistance ratio.

Resistance ratio	Current test (mA)	Reversal rate (s)	Measurements
1 Ω: 1 Ω	50	6	35
100 Ω: 100 Ω	0.5	12	35
10 kΩ: 10 kΩ	0.3	20	35

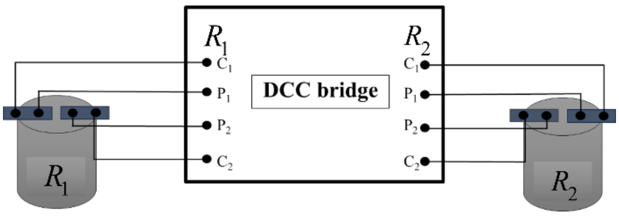

Fig. 2 Cable connections of DCC bridge measurement system

3. Results and Discussion

To evaluate the uncertainty of the ratio error measurement of the DCC bridge obtained from the R-group method, several sources of uncertainty should be considered, such as type A uncertainty, nonlinearity, instrument resolution, resistance standard stability, temperature effect, and power effect. Each of these factors affects the total uncertainty obtained from the analysis, and can be described in a mathematical model, as shown in equation (9):

$$u_c = u_A + u_{nonlin} + u_{res} + u_{sta,R_1} + u_{sta,R_2} + u_{sta,R_3} + u_{TC,R_1} + u_{TC,R_2} + u_{TC,R_3} + u_{P,R_1} + u_{P,R_2} + u_{P,R_3} + u_{uni} \quad (9)$$

Where:

u_A is uncertainty from Type A
u_{nonlin} is uncertainty from nonlinearity
u_{res} is uncertainty from resolution
u_{sta,R_1} is short-term stability of R_1
u_{sta,R_2} is short-term stability of R_2
u_{sta,R_3} is short-term stability of R_3
u_{TC,R_1} is temperature coefficient of R_1
u_{TC,R_2} is temperature coefficient of R_2
u_{TC,R_3} is temperature coefficient of R_3
u_{P,R_1} is power effect of R_1
u_{P,R_2} is power effect of R_2
u_{P,R_3} is power effect of R_3
u_{uni} is uniformity of bath

and show the details of the calculation of the uncertainty source, as shown in Table 2. The type A, short-term stability and power values are as follow: 5.89×10^{-3} μΩ/Ω, 1.00×10^{-2} μΩ/Ω, and 1.44×10^{-5} (for 100 Ω: 100 Ω) and 6.69×10^{-3} μΩ/Ω, 1.00×10^{-2} μΩ/Ω, and 5.20×10^{-4} μΩ/Ω (for 100 Ω: 100 Ω), respectively. The evaluations of combined uncertainty by the R-group method of 1 Ω, 100 Ω, and 10 kΩ ranges are 0.025 μΩ/Ω, 0.022 μΩ/Ω, and 0.023 μΩ/Ω, respectively. In addition, the uncertainty values obtained from the R-group method were compared with the Exchanging method; the comparative results and calculated uncertainty values are summarized in Table 3. When comparing the uncertainty values of the resistance ratios obtained by the two methods, the R-group method yields significantly lower uncertainty than the Exchanging method. The R-group method yields an uncertainty in range of 0.022 μΩ/Ω to 0.025 μΩ/Ω, while the Exchanging method yields a higher uncertainty in rang of 0.031 μΩ/Ω to 0.042 μΩ/Ω. It should be noted that the evaluation of measurement uncertainty by using the Exchanging method can be calculated with the source of standard uncertainty [3]. The R-group method yields a lower overall uncertainty due to the use of three standard resistors in the ratio measurement. This approach reduces the impact of individual variables and allows for a more comprehensive and reliable assessment of data uncertainty. Due to the use of only two standard resistors and the alternating measurement positions by using the Exchanging method, the average and uncertainty estimates are more susceptible to other sources of error than those of the R-group method.

Table 2 Uncertainty budget for the 1 Ω:1 Ω R-group measurement

Uncertainty sources	Probability distribution	Divisor	Sensitivity coefficient	Uncertainty contribution (μΩ/Ω)
Type A	Normal	1	1	1.16 × 10^{-2}
Nonlinearity of the bridge	Rectangular	$\sqrt{3}$	1	5.77 × 10^{-3}
Resolution of the bridge	Rectangular	$\sqrt{3}$	1	2.89 × 10^{-4}
Short-term stability of R_1	Rectangular	$\sqrt{3}$	1	1.00 × 10^{-2}
Short-term stability of R_2	Rectangular	$\sqrt{3}$	1	1.00 × 10^{-2}
Short-term stability of R_3	Rectangular	$\sqrt{3}$	1	1.00 × 10^{-2}
Temperature coefficient of R_1	Rectangular	$\sqrt{3}$	1	2.31 × 10^{-3}
Temperature coefficient of R_2	Rectangular	$\sqrt{3}$	1	2.31 × 10^{-3}
Temperature coefficient of R_3	Rectangular	$\sqrt{3}$	1	2.31 × 10^{-3}
Power of R_1	Rectangular	$\sqrt{3}$	1	1.44 × 10^{-3}
Power of R_2	Rectangular	$\sqrt{3}$	1	1.44 × 10^{-3}
Power of R_3	Rectangular	$\sqrt{3}$	1	1.44 × 10^{-3}
Uniformity of bath	Rectangular	$\sqrt{3}$	1	1.10 × 10^{-2}
Combined standard uncertainty (u_c), coverage factor k=1				0.025

Table 3 Uncertainty values for calculated from R-group method and Exchanging method.

Resistance	R-group	Exchanging
1 Ω:1 Ω		
Combined standard uncertainty, k=1 (μΩ/Ω)	0.025	0.033
100 Ω:100 Ω		
Combined standard uncertainty, k=1 (μΩ/Ω)	0.022	0.031
10 kΩ:10 kΩ		
Combined standard uncertainty, k=1 (μΩ/Ω)	0.023	0.042

To evaluate the potential of the R-group method in the validation and calibration of DCC bridges, the E_n ratio was calculated [7]. The comparison of the E_n ratio values obtained from the R-group method and the Exchanging method in the 1:1 resistance ratio measurement of 1 Ω, 100 Ω, and 10 kΩ using the combined standard uncertainty at the confidence level k = 1 to reduce the influence of unnecessary uncertainty in comparison [8–9], as shown in Equation (10).

$$E_n = \frac{\left|\bar{X}_1 - \bar{X}_2\right|}{\sqrt{u_1^2 + u_2^2}} \quad (10)$$

Where:

$\bar{X}_1$ is Value measured by the R-group method

$\bar{X}_2$ is Value measured by the Exchanging method

u_1 is Measurement uncertainty of the R-group method

u_2 is Measurement uncertainty of the Exchanging method

The E_n ratio values obtained from the R-group method in all ranges are lower than 1 when compared with the Exchanging method and with the uncertainty values of each method. Therefore, it can be concluded that the R-group method can be used for verification and as a guideline for calibration of the DCC bridge appropriately, as shown in Table 4.

Table 4 E_n ratio values in the comparison of measurement methods.

Resistance	Average of ratio error (μΩ/Ω)		E_n ratio
	R-group	Exchanging	
1 Ω:1 Ω	-0.082	-0.088	0.22
100 Ω:100 Ω	-0.032	-0.034	0.06
10 kΩ:10 kΩ	0.061	0.086	0.52

4. Conclusions

This study displays the effectiveness of the R-group method, employing three CCC bridge–calibrated standard resistors, for verifying the ratio error of a commercial DCC bridge model 6010C at the 1:1 ratios of 1 Ω, 100 Ω, and 10 kΩ resistors. The method achieved ratio error values of − 0.082 μΩ/Ω, − 0.032 μΩ/Ω, and 0.061 μΩ/Ω, respectively, which agree with those obtained using the conventional Exchanging method. The combined standard uncertainty (k = 1) was found to be approximately 0.022 μΩ/Ω to 0.025 μΩ/Ω, lower than that of the Exchanging method, thereby improving both the accuracy and stability of the measurement. The calculated En ratios were consistently less than one, confirming the reliability and consistency of the method.

The use of long-term stable standard resistors, calibrated to Quantum Hall standards with uncertainties below 20 ppb, significantly contributes to the confidence in measurement results and the verification process. The findings highlight that employing more than two standard resistors not only mitigates the influence of individual resistor deviations but also enhances measurement stability. Consequently, the proposed method offers a practical and precise alternative for DCC bridge verification in laboratories without access to a CCC bridge. Nonetheless, confirmation against a reliable reference instrument remains essential to ensure the highest accuracy and traceability.

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