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Design of Higher-Order Derivative PID Controller for a Water Tank System

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Abstract

This paper presents an approach to enhance the performance of a water tank control system using a higher-order derivative PID controller (HO-PID) and compares it with a traditional PID controller. The water tank system is modeled as a FOPDT system, and simulations were conducted in MATLAB/Simulink software to evaluate its performance under noise-free and noisy conditions. The simulation results clearly demonstrate that the HO-PID controller, specifically PID_5^5, outperforms the traditional PID controller in all aspects. It achieves superior transient performance with a shorter rise time and settling time, along with zero overshoot. Furthermore, it exhibits excellent disturbance rejection capabilities with the lowest IAE performance index. Under noisy conditions, the PID_5^5 controller demonstrates superior robustness by producing a smoother and less volatile control signal, which is a crucial characteristic for real-world applications.

Keywords: FOPDT, HO-PID, PID, Water Tank System.

1. Introduction

The control of nonlinear systems represents a significant and persistent challenge within industrial process applications. Among these, liquid level control in tanks is a fundamental and ubiquitous problem [1], frequently encountered in sectors ranging from chemical processing to water treatment. These systems are often characterized by inherent nonlinearities and can typically be modeled as first-order plus dead-time (FOPDT) processes [2], [3]. The presence of dead time, in particular, complicates control design, often leading to performance degradation and potential instability. Effectively managing such systems is a key area of research in the control field, demanding robust and efficient control strategies that can ensure stability and optimal performance despite these complexities.

Historically, the Proportional-Integral-Derivative (PID) controller has been the most widely adopted solution for industrial process control, including liquid level regulation. Its popularity stems from its structural simplicity, reliability, and the intuitive nature of its three control terms. However, traditional PID controller [4] exhibit significant limitations, especially when applied to systems with substantial nonlinearity or time delay. These controllers often struggle to provide a satisfactory balance between aggressive tracking and disturbance rejection, and smooth, stable operation. Common drawbacks include excessive overshoot, prolonged settling time, and a lack of robustness to variations in system parameters, which can compromise process efficiency and safety [9].

To address these shortcomings, numerous advanced control strategies have been developed specifically for FOPDT systems. Notable examples include enhanced PID structures such as the Two-Degrees-of-Freedom (2DOF-PID) controller [5], which separates setpoint tracking and disturbance rejection to improve overall performance. The Internal Model Control (IMC) [6] approach has also gained prominence due to its systematic tuning procedure, which directly incorporates the process model to achieve robust and predictable closed-loop responses. Furthermore, other advanced techniques have been applied to tackle these challenges, such as Model Predictive Control (MPC) [7], which uses a system model to predict future behavior and optimize control actions, and Fuzzy Logic Control [8], which relies on a set of rules to handle nonlinearities without requiring a precise mathematical model. However, while these methods offer significant improvements, they often introduce computational complexities or may not fully mitigate the adverse effects of strong dead time and nonlinearities, leaving room for further innovation in control design.

This study is designed to develop and assess the performance of a higher-order derivative PID (HO-PID) controller [9] – [14], which has been recently introduced by Mikulas Huba and his research team. Preliminary findings indicate that this controller significantly enhances response times. Consequently, we have selected this method for the regulation of liquid levels. We begin by approximating the system's mathematical model as a FOPDT system. Building on this foundation, a novel HO-PID controller is designed, which incorporates a second-order derivative term to enhance the system's ability to handle nonlinearities and dead time effectively. To validate the proposed controller's superior performance, the system is simulated in a realistic environment using MATLAB/Simulink. These simulations include tests under system input disturbances and measurement noise. The results are then comprehensively compared against a traditional PID controller, focusing on key metrics such as time-domain performance and the Integral Absolute Error (IAE), thereby confirming the presented HO-PID design's ability to significantly improve system response.

The remainder of this paper is structured as follows: Section 2 details the modeling of the water tank system, including the mathematical model, and the estimation of model parameters. Section 3 focuses on the design of the HO-PID. In Section 4, we present the simulation results using MATLAB/Simulink. Finally, Section 5 provides a conclusion that summarizes the key findings of this research and discusses potential avenues for future work.

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2. Water Tank System

The single tank water system is a fundamental example of industrial process control, illustrating the challenges of managing liquid levels in systems with time-varying dynamics and high nonlinearities. For this study, we utilize a virtual 3D water level control system, simulated in the Factory I/O simulator software [15], as shown in Fig. 1. This system consists of a liquid tank with two control valves and a capacitive level sensor. The control valves are operated by pneumatic actuators that accept control signals ranging from $0V$ to $10V$. The system is primarily intended for level and flow control using a PID controller. The physical properties of the tank are as follows: a height of $3m$, a diameter of $2m$, a discharge pipe radius of $0.125m$, a maximum input flow of $0.25m^3/s$, and a maximum output flow of $0.3543m^3/s$ [15].

This section will detail the mathematical modeling of this system, its linearization, and the estimation of its FOPDT model parameters.

Fig. 1 Water tank system in Factory I/O simulator [15].

2.1 Mathematical Model for Water Tank System

The dynamic behavior of a single water tank system, as illustrated in the provided Fig. 2, can be described by a nonlinear differential equation. This equation is derived from the principle of mass balance, where the rate of change of the water volume in the tank equals the inflow rate minus the outflow rate.

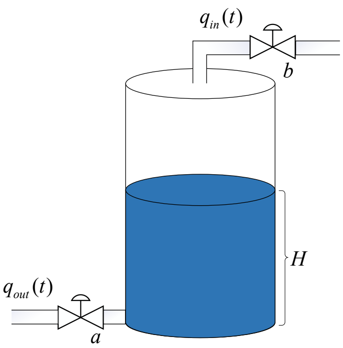

Fig. 2 Sigle tank system model.

A fundamental principle for modeling a single water tank system is the conservation of mass, which states that the rate of change of liquid volume inside the tank is equal to the inflow rate minus the outflow rate. Based on this principle, the fundamental equation for the rate of change of the water level in the tank can be expressed as

$$A\frac{dH}{dt} = q_{in}(t) - q_{out}(t)\,, \tag{1}$$

where A is the cross-sectional area of the tank, H is the water level, $q_{in}(t)$ is the inflow rate, and $q_{out}(t)$ is the outflow rate. According to Torricelli's Law, the outflow rate is dependent on the water level, which introduces a nonlinear term into the system's equation:

$$q_{out}(t) = a\sqrt{H}\,, \tag{2}$$

where a is the outflow constant. Similarly, the inflow rate is expressed as $q_{in}(t) = bV$, where b is the inflow constant and V is the control input. Hence, the nonlinear dynamic equation describing the water tank system is given by

$$A\frac{dH}{dt} = bV - a\sqrt{H}\,. \tag{3}$$

2.2 Estimation of the FOPDT Model

The FOPDT model [2] is a widely used mathematical representation in control engineering, particularly for industrial processes that exhibit first-order dynamics and a time delay. This model is favored because of its structural simplicity and its ability to accurately reflect the dynamic behavior of many real-world systems.

From the linearized differential equation derived in the previous section, the system's dynamics can be represented by the following FOPDT transfer function:

$$G_p(s) = \frac{K_p e^{-L_p s}}{T_p s + 1}\,, \tag{4}$$

where K_p is process gain, T_p is time constant, and L_p is dead-time.

The modeling procedure was conducted as follows. Experimental data were collected from the water level control system using the Factory I/O environment. During the test, the outlet valve was fixed at 50%, and the inlet valve was also set to 50%. The experiment was run for $500s$. The measured input was the control voltage (in volts), and the output was the water level (in meters). These input-output datasets were then used to estimate a mathematical model via the System Identification Toolbox in MATLAB. The identified parameters of the FOPDT model were: $K_p = 0.2982$, $T_p = 70.448s$ and $L_p = 1.8s$.

Based on these parameters, the transfer function of the system can be expressed as:

$$G_p = \frac{0.2982}{70.448s + 1} \cdot e^{-1.8s}\,. \tag{5}$$

To validate the accuracy of the identified model, the step response of the FOPDT transfer function was compared with the actual system response obtained from the experiment. As shown in Fig. 3, the model response (dashed line) closely matches the measured data, with a goodness of fit of 95.9%. This result confirms that the simplified FOPDT model effectively captures the essential dynamic behavior of the water level system, particularly during the transient and steady-state phases.

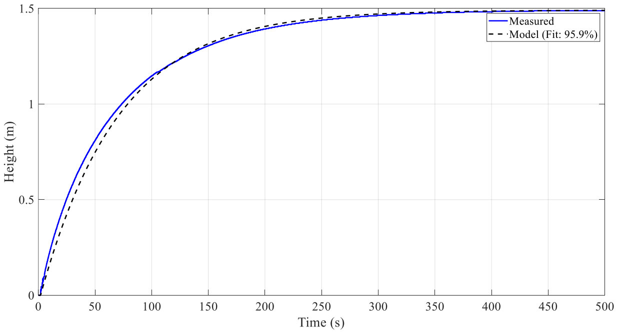

Fig. 3 Comparison between the tank level response and the first-order process model.

3. Higher-Order Derivative PID Controller

The HO-PID or PID_n^m controller was developed by Mikulas Huba and his research team [9, 10, 11, 13, 14, 16], evolving from the traditional PID controller. It incorporates additional higher-order derivative terms and filters to enhance control performance.

The PID controller with the degree of derivative being added to PID^m [16] contains higher degree of derivative and can be defined as:

$$C^m(s) = K_c + \frac{K_i}{s} + K_{D_1}s + \ldots + K_{D_m}s^m\,, \tag{6}$$

where $m = 0, 1, 2, \ldots$.

The purpose of the prefilter $F_p(s)$ in the control circuit is to prevent overshoot and to accelerate the system's response. Overshoot is avoided by designing the prefilter denominator to cancel the numerator of the setpoint-to-output transfer function. The response is sped up by the design of the prefilter numerator.

$$F_p(s) = \frac{b_{m+1}T_iT_{D_m}s^{m+1} + \ldots + b_2T_iT_{D_1}s^2 + b_1T_is + 1}{T_iT_{D_m}s^{m+1} + \ldots + T_iT_{D_1}s^2 + T_is + 1}\,, \tag{7}$$

where $m = 0, 1, 2, \ldots$.

An ideal controller that for production requires a additional filter, so the control is expanded by the n-order binomial filter.

$$Q(s) = \frac{1}{(T_fs+1)^n}\,, \tag{8}$$

where $n = 0, 1, 2, \ldots$ and $n \geq m$.

So, this controller can be described as follows:

$$C_n^m(s) = PID^m(s)Q_n(s)\,. \tag{9}$$

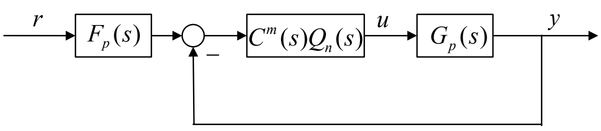

Fig. 4 Block diagram of the feedback control with HO-PID controller.

Based on the HO-PID controller structure shown in Fig. 4, an example of parameter tuning for the PID_n^m $(m = 0)$ with a prefilter $F_p(s)$ is presented using the relationships in (10),

$$C(s) = K_c\frac{1+T_is}{T_is} = K_c + \frac{K_i}{s};\; F_p(s) = \frac{b_1T_is+1}{T_is+1}\,. \tag{10}$$

The system's behavior is described by the transfer function:

$$\frac{Y(s)}{R(s)} = \frac{K_pK_c(b_1T_is+1)}{T_is(T_ps+1)e^{L_ps} + K_pK_c(T_is+1)}\,. \tag{11}$$

This function yields the characteristic quasi-polynomial shown in (12),

$$P(s) = T_is(T_ps+1)e^{L_ps} + K_pK_c(T_is+1)\,. \tag{12}$$

The optimal set of parameters, K_c and T_i along with the prefilter tuning b_1 are determined using the Triple Real Dominant Pole (TRDP) method. This method requires identifying a dominant pole, s_0, that generates a triple root in the characteristic equation, subject to the conditions $P(s_0) = 0$, $\dot{P}(s_0) = 0$, and $\ddot{P}(s_0) = 0$. The detailed derivation of these parameters is provided in (13) – (16),

$$s_o = -\frac{A+4-S}{2L_p}\,, \tag{13}$$

where $A = \dfrac{L_p}{T_p}$ and $S = \sqrt{A^2+8}$,

$$K_c = \frac{(S-2)e^{(S-A-4)/2}}{K_pL_p}\,, \tag{14}$$

$$T_i = \frac{2(2-S)L_p}{A^2+2A+28-(A+10)S}\,, \tag{15}$$

$$b_1 = \frac{1}{T_is_o} = \frac{A^2+2A+28-(A+10)S}{(S-2)(S-A-4)}\,. \tag{16}$$

4. Simulation Results

To evaluate the performance of the HO-PID controller, a water tank system was simulated using a process approximated by a FOPDT model. The system parameters were identified as follows: process gain $K_p = 0.2982$, time constant $T_p = 70.448s$, and dead-time $L_p = 1.8s$. The simulations were conducted using MATLAB/Simulink with a sample time of $0.1s$ and a total simulation duration of $2000s$.

This study compares the performance of two controllers: (i) the traditional PID controller tuned using the IMC method with a tuning parameter of $\lambda = 20$, selected to minimize the IAE and (ii) the HO-PID controller, designed with a maximum derivative order of $m \leq 5$ and a binomial filter order of at $n = 5$ at $T_f = 16.5$. To simplify, we define a prefilter using a binomial filter of $n = 2$ as described in [14]. The controller parameters for the HO-PID are listed in Table 1.

The simulation was conducted under two scenarios: one without measurement noise and the other with noise. The first interval, from $0s$ to $1000s$, evaluated the system's step response to a reference level of $1.5m$. The second interval, from $1000s$ to $2000s$, assessed the controllers' ability to reject input disturbances.

To quantitatively assess performance, time-domain response metrics (namely rise time, settling time, and percentage overshoot) were measured. In addition, the IAE performance index was used to evaluate the overall control effectiveness of each controller.

Table 1. HO-PID controller parameters.

	K_c	K_i	K_{D_1}	K_{D_2}	K_{D_3}	K_{D_4}	K_{D_5}
PID_5^0	1.858	0.023	-	-	-	-	-
PID_5^1	4.269	0.038	0.112	-	-	-	-
PID_5^2	7.849	0.062	0.304	0.039	-	-	-
PID_5^3	14.27	0.103	0.662	0.133	0.102	-	-
PID_5^4	30.24	0.206	1.566	0.385	0.464	0.226	-
PID_5^5	143.7	0.939	8.021	2.225	3.324	2.571	8.110

Remark: $K_{D_1}\times10^3$, $K_{D_2}\times10^5$, $K_{D_3}\times10^6$, and $(K_{D_4}, K_{D_5})\times10^7$

4.1 Simulation Results without Noise

Based on the simulation results shown in Fig. 5 and Table 2 under noise-free conditions, the HO-PID controllers (specifically PID_5^4 and PID_5^5) demonstrate significantly better reference tracking performance than the traditional PID controller. These higher-order controllers achieve shorter rise time and lower percentage overshoots. As illustrated in Fig. 5(a), increasing the derivative order in the HO-PID structure enhances the system's ability to track a $1.5m$ step reference with improved speed and accuracy. The corresponding control signals in Fig. 6(b) for the PID_5^5 controller are smoother and exhibit fewer oscillations.

Furthermore, Fig. 5(c) presents the system's response to an input disturbance applied from $1000s$ to $2000s$. The PID_5^5 controller demonstrates a superior disturbance rejection capability compared to both the traditional PID and lower-order HO-PID controllers, maintaining the water level near the desired setpoint with greater stability. T0his is quantitatively supported by the IAE metric in Table 2, where PID_5^5 achieves the lowest overall IAE of 54.86. Notably, under the disturbance scenario, the IAE value drops to 1.06, highlighting the controller's high precision and effectiveness in minimizing output error.

4.2 Simulation Results with Noise

This section presents the simulation results showing the performance of the controllers when the system is subjected to noise. A band-limited white noise block was used, with a noise power of 0.001 and a sample time of $4s$. As shown in Fig. 6(a) and Table 3, the HO-PID controllers, particularly PID_5^5, demonstrate superior reference tracking. They achieve significantly shorter rise time and lower overshoots compared to the traditional PID controller. The values in Table 3 also confirm that the HO-PID can effectively reduce the IAE during the step response, reflecting higher control accuracy. The corresponding control signals in Fig. 6(b) show that PID_5^5 produces a smoother and less volatile signal than the other controllers, even with measurement noise present.

For the disturbance rejection tests in Fig. 6(c), conducted between $1000s$ to $2000s$, the PID_5^5 controller again demonstrates the best performance by maintaining the water level near the reference value with significantly less fluctuation. Fig. 6(d) further illustrates that the control signal of PID_5^5 is less volatile under these conditions, confirming its superior ability to handle disturbances. When evaluating the quantitative performance indices in Table 3, the PID_5^5 controller yields the lowest IAE value of 1.06, which are the best results among all controllers tested. This confirms that the HO-PID controller possesses superior robustness to noise compared to the traditional PID and lower-order HO-PID controllers.

Table 2. Performance of simulation without noise.

Controllers	**Rise time**	**Settling time**	**%Overshoot**	**IAE**
		Step responses		
PID	80.1086	131.8111	0.3936	58.41
PID_5^0	160.9709	504.1695	5.9827	296.5
PID_5^1	100.7701	304.4883	5.9180	195.1
PID_5^2	73.5806	201.6114	4.1151	137.4
PID_5^3	58.9747	111.2366	0.4135	98.65
PID_5^4	54.5538	113.4650	0.0000	73.89
PID_5^5	55.3741	99.8267	0.0000	54.86
		Disturbance rejection		
PID	-	164.9482	5.2028	11.27
PID_5^0	-	322.7042	14.3523	45.80
PID_5^1	-	221.8293	11.6776	26.10
PID_5^2	-	164.4332	9.0821	16.30
PID_5^3	-	119.6235	6.4872	9.75
PID_5^4	-	65.7452	3.8129	4.85
PID_5^5	-	0.0000	0.9888	1.06

Table 3. Performance of simulation with noise.

Controllers	**Rise time**	**Settling time**	**%Overshoot**	**IAE**
		Step responses		
PID	79.4524	131.7789	0.7818	60.43
PID_5^0	160.6506	501.0646	5.9095	297.2
PID_5^1	100.4719	304.8091	6.0075	197.0
PID_5^2	73.0668	201.7164	4.1154	139.4
PID_5^3	58.3524	109.8072	0.6301	101.8
PID_5^4	53.5310	112.3676	0.9129	78.24
PID_5^5	55.3741	99.8267	0.0000	54.86
		Disturbance rejection		
PID	-	159.8414	5.3220	12.79
PID_5^0	-	319.8406	14.3991	45.80
PID_5^1	-	217.2324	11.7912	27.40
PID_5^2	-	158.8933	9.2291	17.90
PID_5^3	-	121.3039	6.5218	11.60
PID_5^4	-	73.7834	3.6745	7.45
PID_5^5	-	0.0000	0.9888	1.06

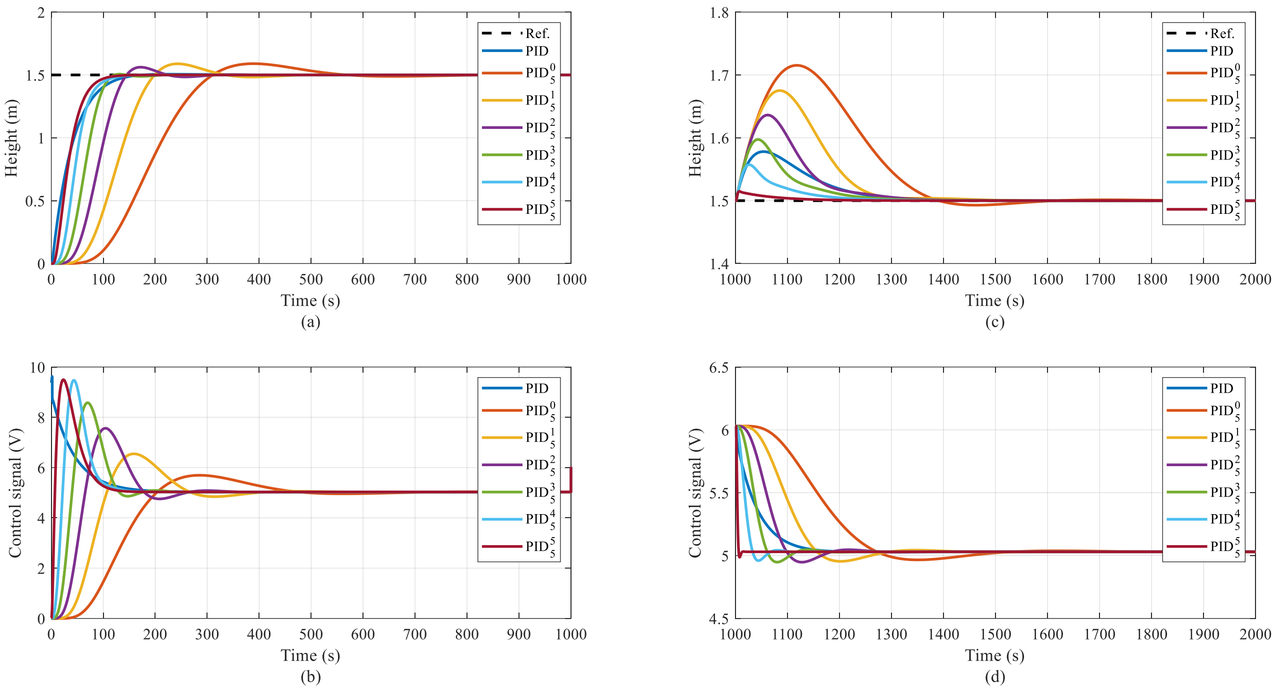

Fig. 5 Simulation results of a water level control system under noise-free conditions, comparing the performance of traditional PID and HO-PID. (a) System's step response during setpoint tracking to a reference of 1.5 m. (b) Corresponding control signals generated during the tracking phase. (c) System response to an input disturbance. (d) Control signals applied during the disturbance rejection phase.

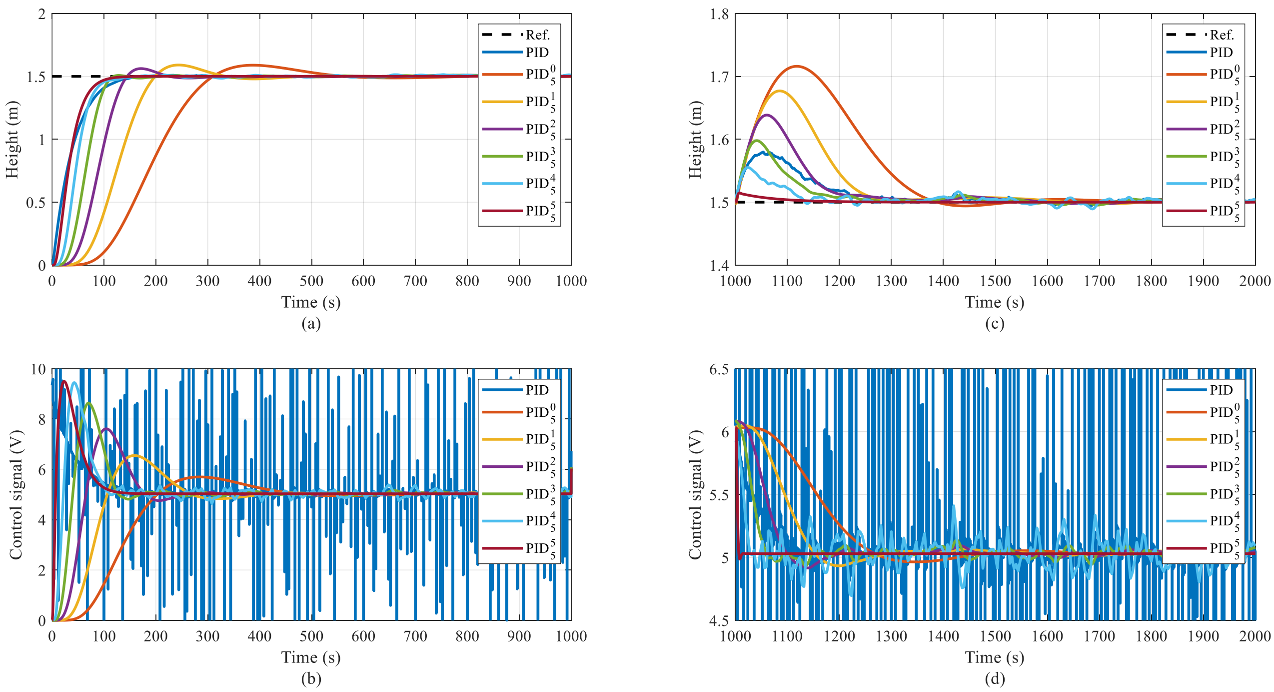

Fig. 6. Simulation results of a water level control system under noisy conditions, comparing the performance of traditional PID and HO-PID controllers. (a) Step response for setpoint tracking in the presence of measurement noise. (b) Corresponding control signals during the tracking phase. (c) System response during input disturbance rejection. (d) Control signals during disturbance rejection under noisy conditions.

5. Conclusion

This research investigated and analyzed the performance of a water level control system using HO-PID controller in comparison to a traditional PID controller. The study's findings can be summarized as follows. This work proposed an enhancement to the traditional PID controller by incorporating higher-order derivative terms into its structure, aiming to improve the control performance of sluggish and high-inertia systems like the water level system. From the simulations conducted under both noise-free and noisy conditions, it was found that the HO-PID controller, particularly the PID_5^5, consistently demonstrated a superior performance to the traditional PID controller. Under noise-free conditions, the PID_5^5 controller achieved a faster and more accurate response to the reference signal, characterized by a shorter rise time and a lower overshoot percentage. Furthermore, it exhibited excellent disturbance rejection capabilities, maintaining the water level near the setpoint with great stability, which was supported by the lowest quantitative performance index, IAE. When simulated under noisy conditions, the results further confirmed the superior robustness of the PID_5^5. It effectively managed noise, resulting in a smoother and less volatile control signal, a crucial characteristic for real-world applications. In conclusion, the application of a HO-PID controller, specifically the PID_5^5, can significantly enhance the performance of a water tank control system in terms of transient response, disturbance rejection, and control accuracy. This research supports the notion that the HO-PID concept is a highly effective alternative for application in industrial process control systems.

For future work, further research should involve applying the HO-PID controller to a real physical system to validate its performance in a real-world scenario.

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