

Variable Step-Size Indirect Frequency Estimation Algorithm Based On Accumulative Gradient

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Abstract

A variable step-size indirect frequency estimation (VIFE) algorithm is proposed in this paper. The step-size of IFE is adjusted so as to improve the convergence properties by using an accumulative gradient technique. Simulation results have been drawn to demonstrate the superiority of the proposed VIFE.

Keywords: Adaptive algorithm, Adaptive signal processing, Adaptive notch filter, Frequency estimation

1. Introduction

Estimating the frequency of sinusoidal signals is a challenging problem and can be found in applications in many fields, such as data communication systems [1], smart grid systems [2], radar [3], and so on. Studies that present frequency estimation are many. But we will only be looking at IFE [4] in this article. As has been mentioned in [4], the IFE is marginally better than the compared algorithms in computation at the cost of some degradation in convergence; the convergence properties of the IFE must be further improved. IFE sets step-size μ as a constant value, making it impossible to control the convergence properties. This problem is therefore the motivation for this work.

To improve the convergence properties of the IFE, we propose a new technique to adjust value of step-size μ . In the early state of adaptation, it is set to have a high value, and after convergence, it is set to have a low value. By doing this, the VIFE will converge faster and provide more accurate solutions.

2. Proposed Technique

In this work, a second order adaptive FIR notch filter [4] is used to estimate indirectly the frequency of a noise-embedded sinusoidal signal. The transfer function is defined by

$$H(z) = 1 + az^{-1} + z^{-2} \quad (1)$$

where $-2 < a < 2$ is the notch parameter to be adapted. The relationship between a and notch frequency Ω_N is $a_N = -2 \cos \Omega_N$. The input signal to $H(z)$ is assumed to be a single frequency sinusoidal signal embedded in noise of the form

$$x(n) = A \cos(\Omega_0 n + \phi) + v(n) \quad (2)$$

where $A > 0$ is the amplitude, $0 < \Omega_0 < \pi$ is the input frequency, $-\pi \leq \phi \leq \pi$ is the phase and $v(n)$ is an additive white Gaussian noise with zero mean and variance of σ^2 . An input signal to noise ratio is defined by $\text{SNR} = A^2 / 2\sigma^2$. The notch filter $H(z)$ is implemented by using the direct form I structure to produce the error signals $e(n)$ as follows

$$e(n) = x(n) + ax(n-1) + x(n-2). \quad (3)$$

It is assumed that A, Ω_0 and ϕ are unknown constants. In this work, however, we are focusing only on estimating Ω_0 . To estimate Ω_0 , the IFE adaptive algorithm [4] is utilized which is defined by

$$a(n+1) = a(n) - \mu e(n) \{s(n) - a(n)x(n)\} \quad (4)$$

where μ is the step-size and is a positive fixed value by definition

$$s(n) = x(n-1). \quad (5)$$

Now let us introduce a time varying step-size $\mu(n)$ as follows:

$$\mu(n+1) = \alpha \mu(n) + \beta p^2(n) \quad (6)$$

where

$$p(n) = \alpha p(n-1) + \beta g(n-1), \quad (7)$$

$$g(n) = e(n)s(n) \quad (8)$$

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$0 < \alpha < 1$, $0 < \{\gamma, \beta\} < 1$, and $g(n)$ is known as a gradient signal. Eq. (7) is an accumulative value of the gradient signal $g(n)$ which is used to adjust the step-size as shown in Eq. (6). Ultimately, the proposed VIFE is derived as

$$a(n+1) = a(n) - \mu(n)e(n)\{s(n) - a(n)x(n)\} \quad (9)$$

In the next section, the performance of the proposed VIFE will be discussed.

3. Numerical Results

In this section, the performances of the proposed VIFE are demonstrated and compared with those of IFE. Fig. 1 shows the averaged values of the proposed time-varying step-size parameter for 5 and 10 dB of SNR. The parameters setting herein include $SNR = \{5, 10\}$ dB, $\Omega_0 = 0.1\pi$, $\phi \in [-\pi, \pi]$, $\mu(0) = 0$, data length $L = 10\,000$ and 100 computer runs. It is seen that at the early stage of adaptation, the step-size is high. After converging, its value is low as desired. Therefore, it ensures that the convergence of the IFE is improved, which is shown in Fig. 2.

In Fig. 2, the proposed and comparative algorithms are assigned to estimate the input signal frequency under the following conditions: $SNR = \{5, 10\}$ dB, $\phi \in [-\pi, \pi]$, $\Omega_0 = 0.1\pi$, $a(0) = 0$, $\mu(0) = 0$, @5dB $\mu_{ife} = 0.012$, @10dB $\mu_{ife} = 0.0012$, data length $L = 10\,000$ and 100 computer runs. The step size μ of the IFE is adjust to yield the same simulated mean square error (MSE) as those of the proposed VIFE. The general form of the MSE is calculated by

$$MSE = \text{var}(a) + \text{bias}^2 \quad (10)$$

where $\text{var}(a)$ is the variance of $a(n)$, $\text{bias} = a_0 - E[a(n)]$, and $a_0 = -2\cos\Omega_0$. To obtain the estimated MSE, the 100 independent runs of each algorithm are ensemble averaged. The results are shown in Fig. 2. It is seen that at a low SNR scenario (5 dB), the performances of both algorithms are identical, whereas at a high SNR (10 dB) the proposed VIFE shows faster convergence speed than the conventional IFE adaptive algorithm.

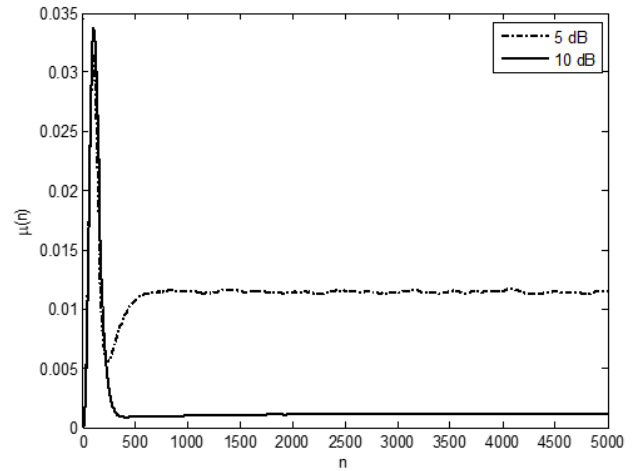


Fig. 1. Proposed time varying step-size parameter for 5 and 10 dB of SNR

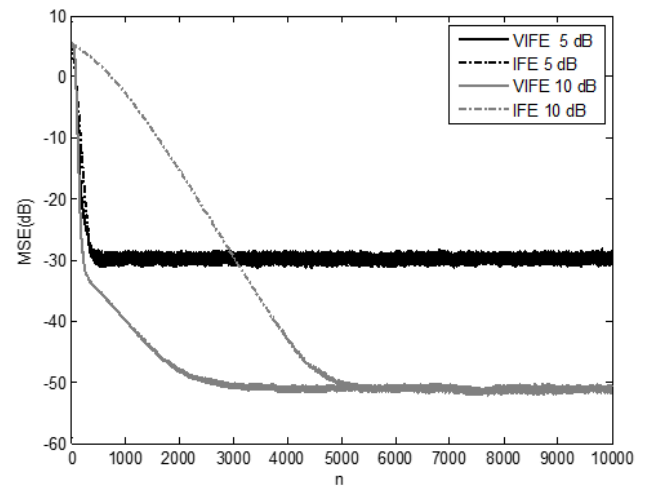


Fig. 2. Convergence speed comparison at the same value of MSE for 5 and 10 dB of SNR.

4. Conclusions

A VIFE adaptive algorithm for a second-order adaptive FIR notch filter for single frequency estimation is proposed. As compared to the conventional IFE, VIFE shows better convergence speed in a high SNR value scenario. Steady state analysis of the proposed VIFE in an interesting topic and will be done in the further work.

References

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