Finite Precision Investigation of OFDM Implementation

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Abstract

This paper focuses on the development of precision hardware for digital signal processing using fixed-point representation for the implementation of orthogonal frequency division multiplexing (OFDM). The primary emphasis is on the fast Fourier transform (FFT), which is essential for the transmitter, as it converts signals from the time domain to the frequency domain. Additionally, the paper discusses linear interpolation, a technique used to acquire the channel response for data recovery. Both FFT and linear interpolation demand strong computational resources from hardware, and they greatly influence the bit error rate (BER). The MATLAB numerical simulation results are employed, and it indicates that the optimal precision for the fixed-point representation is between 10 and 13 bits, which is a favorable balance between system performance, hardware complexity, and power consumption.

Keywords: OFDM, FPGA, Fast Fourier Transform, Linear Interpolation

1. Introduction

Orthogonal frequency division multiplexing (OFDM) has become a fundamental part of modern wireless communication systems because of its ability to minimize multipath fading and enhance bandwidth efficiency effectively. The fast Fourier transform (FFT) is the primary processing component in the OFDM receiver architecture and allows signal processing in the frequency domain. However, the accuracy of FFT-based transforms is susceptible to distortions caused by factors like time and frequency offsets, especially in environments with rapidly changing channels [1].

Channel estimation plays a pivotal role in ensuring reliable signal recovery in OFDM systems. Typically, linear interpolation methods are used to estimate the channel impulse response (CIR) through least squares (LS) or minimum mean square error (MMSE) techniques [2]. Nevertheless, these approaches often prove insufficient to capture the subtle temporal and spectral fluctuations intrinsic to dynamic wireless channels.

For hardware implementation, like a field programmable logic array (FPGA), a fixed-point number system is mainly used to represent the calculation and computation format. The question is what the optimum bit error rate (BER) performance is for both signal transformations via FFT and channel estimation in OFDM systems. This work aims to enhance and report the estimation accuracy and processing reliability under fast-fading and frequency-selective conditions for high speed communication systems. The study further explores the mathematical implications of fix-points and demonstrates their applicability through simulation results. Ultimately, this work seeks to contribute to the development of high-precision OFDM-based communication architectures, with potential applications in next-generation wireless systems that demand resilience and adaptability under complex propagation scenarios. Additionally, we also investigate the trade-off between energy consumption and system performance by varying the fractional bit allocation in an 18-bit fixed-point OFDM system. Through extensive simulations, we analyze the BER performance under different quadrature amplitude modulation (QAM) orders and in the presence of frequency selective fading channels. Our study aims to provide valuable insights into the design of energy-efficient OFDM systems that maintain acceptable levels of performance, particularly in the context of fixpoint arithmetic, where the impact of finite word length and fractional bit representation can be significant.

2. Overview of the OFDM system

2.1 Basic of OFDM system

The OFDM system consists of a transmitter and a receiver, as illustrated in Fig.1. The input bit stream is first mapped into symbols, where the QAM modulation method is generally used. Next, the symbols stream are converted from serial to parallel. Then, the pilot symbols are inserted to acquire the channel response. The frequency domain symbols are converted into a sequence of the time domain symbols by the inverse FFT (IFFT) process. The IFFT calculation is given by [3]

$$x(n)=\frac{1}{\sqrt{N}}\sum_{k=0}^{N-1}X(k)e^{\left(\frac{2j\pi kn}{N}\right)} \quad n=0,1,2\ldots N-1, \quad (1)$$

where $X(k)$ is QAM signal in frequency-domain signal, $x(n)$ is OFDM signal in time-domain signal. N is the total of subcarrier. To eliminate interference (ISI), a cyclic prefix (CP) is added to the front of each OFDM symbols, and then the symbols are converted from parallel to serial. At the receiver end, the signal is ultimately transmitted across CIR, denoted by $h(n)$, and the Additive White Gaussian Noise (AWGN), $z(n)$ is considered. The received signal calculation is obtained by

$$y(n)=x(n)*h(n)+z(n), \quad (2)$$

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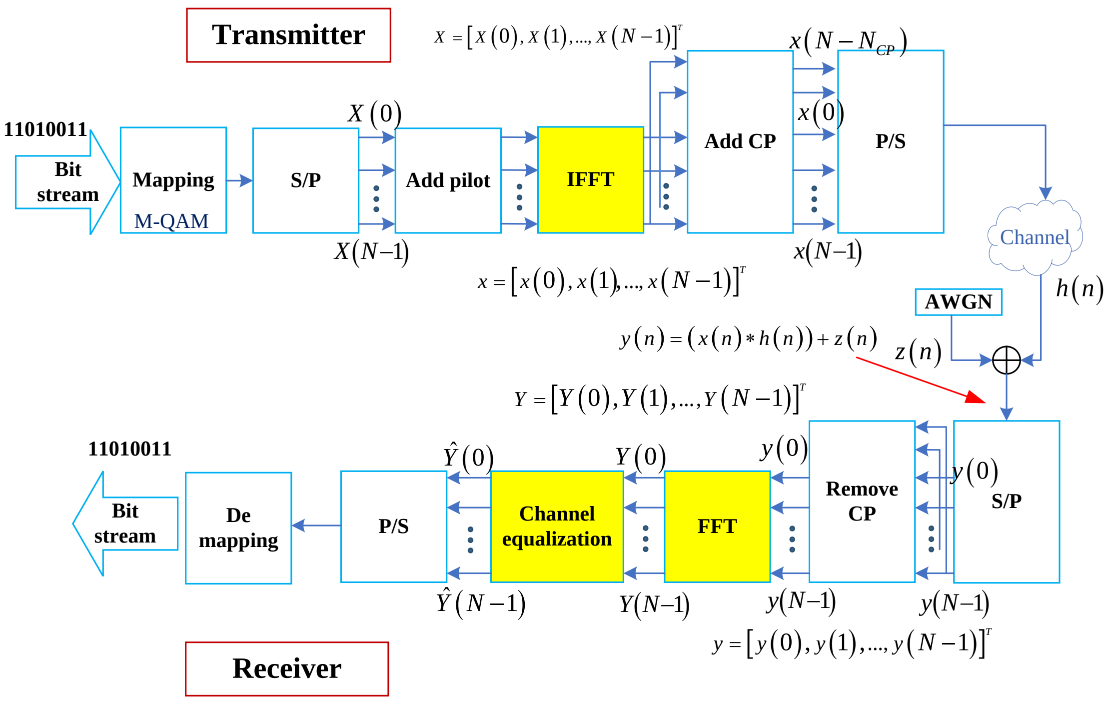

Fig. 1 Block diagram of considered OFDM system processing unit.

where * is convolution operation. The revived signal stream is converted from a serial to a parallel format, and the cyclic prefix (CP) is eliminated. Afterward, the signal is converted from time to frequency domain by the process FFT can be computed by,

$$X(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} x(n) e^{\left(\frac{-j2\pi kn}{N}\right)} \quad k = 0,1,2\ldots,N-1. \quad (3)$$

Finally, the signal is converted from parallel to serial format and the bits are separated to return to the original bit signal.

2.2 Spilt Radix Calculation

The IFFT and FFT are the primary algorithms in the field of digital signal processing (DSP) for OFDM processing. IFFT/FFT is a fast calculation of discrete Fourier transform (DFT). DFT is difficult to compute in the real world. The IFFT/FFT algorithm reduce the complex multiplication, and, it can be easily implement in the real world. The calculation pair is express as [4],

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X(k) W_N^{-kn}, \quad (4)$$

and

$$X(k) = \sum_{k=0}^{N-1} x(n) W_N^{kn}. \quad (5)$$

Equation (4) is IFFT and (5) is FFT equation, respectively. W_N^{kn} is Twiddle factor.

IFFT/FFT algorithm is included of Radix-2, Radix-4, and Spilt Radix. Spilt Radix is used in this work because it has a small number of complex multiplications and fewer stages than Radix-2 and Radix-4.

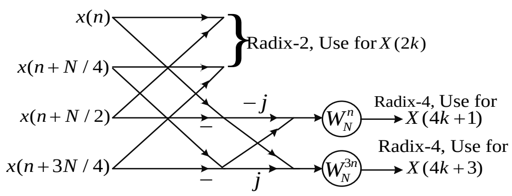

Fig 2. Split-Radix "L" shaped

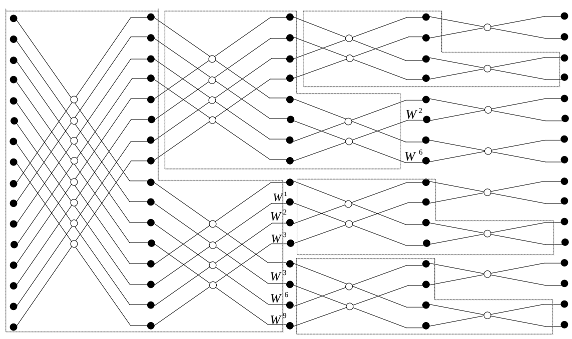

Fig 3. Butterfly diagram Split-Radix FFT for 16 points.

In Split-Radix, it split N into two part, which are $N/2$, and $N/4$. Split-Radix algorithm combines the simplicity of Radix-2 and Radix-4. Where Radix-2 is used for the even input index, output is $X(2k)$ as shown in (6), and Radix-4 is used for the two odd input index and output is $X(4k+1)$ and $X(4k+3)$ as shown in (7) and (8), As can be see, the butterfly is in "L"- shaped, as shown in Fig 2.

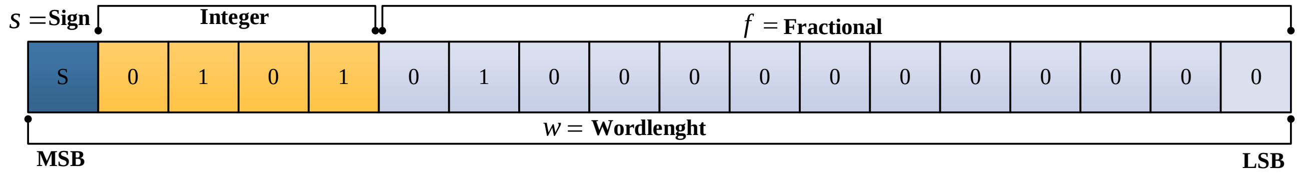

Fig. 4. Fixed-point number.

$$X(2k)=\sum_{n=0}^{N/2-1}\left[x(n)+x\left(n+\frac{N}{2}\right)\right]\cdot W_{N/2}^{nk}\,, \qquad (6)$$

$$X(4k+1)=\sum_{n=0}^{N/4-1}\left\{\left[x(n)-x\left(n+\frac{N}{2}\right)\right]-j\left[x\left(n+\frac{N}{4}\right)-x\left(n+\frac{3N}{4}\right)\right]\right\}\cdot W_N^n W_{N/4}^{3k} \qquad (7)$$

$$X(4k+3)=\sum_{n=0}^{N/4-1}\left\{\left[x(n)-x\left(n+\frac{N}{2}\right)\right]+j\left[x\left(n+\frac{N}{4}\right)+x\left(n+\frac{3N}{N}\right)\right]\right\}\cdot W_N^{3n} W_{N/4}^{kn} \qquad (8)$$

From (6) – (8), we can extend to butterfly diagram Split-Radix FFT used for 16 points shown as in Fig 3. By (3), we get the received signal pair in frequency domain, denote by $Y\ k$ ..

2.3 Fixed-point Numbers

In this paper, Fixed-point numbers [5] is used. The fraction bits are investigated, which is considered in numerical simulation analysis. The fractional bits can be specified during the system design process. In FPGA, the maximum word length is 18 bits, which can be specified as shown in Fig 4. w is word length; and in this paper, 18 bits is used, s represents the format sign 1 bit, f is fractional 13 bits. In this case, integer can be computed by $w-(f-s)$ bits, in this paper integer of 4 bits is employed.

3. Interpolation in OFDM system

At receiver after FFT can be rewritten in the frequency domain as follows (9)

$$Y(k)=X(k)H(k)+Z(k), \qquad (9)$$

where $Y(k)$ is OFDM signal within pilot signal, $X(k)$ is Data receiver signal, $H(k)$ is channel communication in received, and $Z(k)$ is noise signal (AWGN). Linear interpolation method employed, simple to implementation, can be seen in (10) [6].

$$\hat{H}_{Li}(k)=\hat{H}_{Li}(mL+l)$$
$$=\left(\hat{H}_{LS}(m+1)-\hat{H}_{LS}(m)\right)\frac{l}{L}+\hat{H}_{LS}(m), \qquad (10)$$

when $\hat{H}_{Li}(k)$ is the estimate of the channel between the linear intervals with subcarrier k, L is the length of the interval between the pilot signals $m=0,1,\ldots N_P-1$ and $l=0,1,\ldots L-1$.

The channel estimated by using LS can be seen in (11), which is calculated as,

$$\hat{H}_{LS}(k)=\frac{Y(k)}{X_P(k)}, \qquad (11)$$

when $\hat{H}_{LS}(k)$ is an approximation using the LS method, $Y(k)$ is the received frequency signal, $X_P(k)$ is pilot signal, which is known by the receiver.

When received $\hat{H}_{LS}(k)$ from (11) is estimated, we can obtain estimated $\hat{Y}(k)$ signal after channel equalization as follows

$$\hat{Y}(k)=\frac{X(k)H(k)}{\hat{H}_{LS}(k)}+\frac{Z(k)}{\hat{H}_{LS}(k)}$$
$$=X(k)+\frac{Z(k)}{\hat{H}_{LS}(k)}, \qquad (12)$$

hence, $\hat{H}_{LS}(k)$ is channel estimate using LS, $\hat{Y}(k)$ is an approximation of the received signal after equalized, $X(k)$ is QAM symbols.

4. Simulation result

In this section, the description of the BER performance in OFDM processing using fixed-point calculation is proposed, where the simulated parameters are in Table 1.

Fig. 5 comparison of the performance of different QAM modulation formats for fractional bits, using the parameter values from Table 1. The fractional bit values considered are 10, 12, and 14 bits, with a SNR of 35 dB. The X-axis, labeled "Vary QAM" and annotated with numerical values (4, 64, 256, 1024), which likely represent different QAM orders. A precise definition of "Vary QAM" on the X-axis would help readers accurately understand how changes in QAM order affect BER performance for each fractional bit. Given the trends of the lines, the fractional bits 12 proves the lowest BER and the best fractional bits performance is between 10 and 12 bits, and specifying the range of QAM orders where 12 bits provide optimal performance would enhance the accuracy of the analysis.

TABLE I. Setting parameters on OFDM system

Parameter	Setting
Subcarrier	4096
Symbols	14
Modulation	4,16,64,256,1024-QAM
No. of CP	1024
SNR	0-40 dB
IFFT/FFT	4096

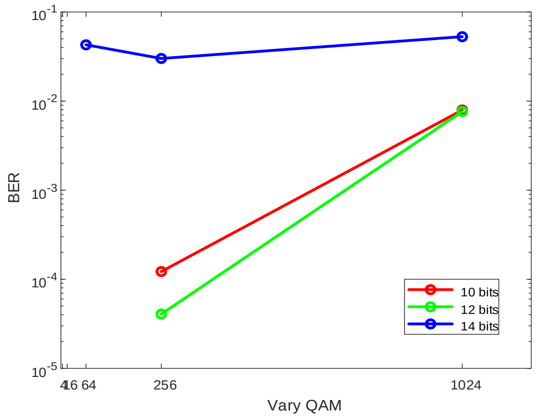

Fig. 5. The performance comparison of BER versus vary QAM by setting SNR 35 dB

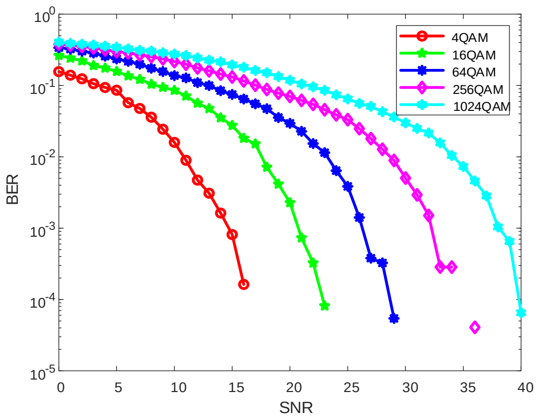

Fig. 6. The performance comparison of BER versus SNR by using fractional 13 bits

Fig. 6 illustrates the results of a MATLAB simulation. This simulation was designed to evaluate the performance of various QAM schemes specifically 4-QAM, 16-QAM, 64-QAM, 256-QAM, and 1024-QAM in terms of BER versus SNR. The simulation incorporated a 4096-point IFFT and FFT utilizing a Split-Radix algorithm with 13 fractional bits. The X-axis of the graph represents the SNR, while the Y-axis displays the corresponding BER.

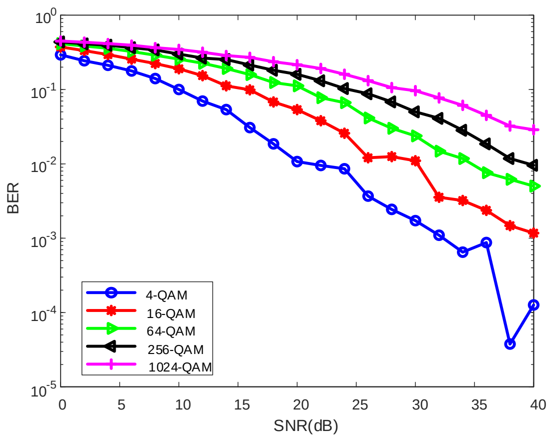

Fig.7 QAM-modulation scheme with channel estimation by linear interpolation with 10 OFDM symbols.

Fig. 7 illustrates the simulated performance of an OFDM system using the Linear Interpolation, specifically for various levels of QAM complexity ranging from 4-QAM to 1024-QAM. From the graph, it is evident that the results for BER versus SNR do not follow the expected linear relationship seen in an ideal channel. The primary reason for this non-linearity is that the OFDM signal is significantly affected by a multipath channel, and sensitivity to subcarriers and channel conditions. In this simulation, which uses 10 OFDM symbols, the results clearly show that as the modulation order increases (from 4-QAM to 1024-QAM), the BER curve degrades (higher BER for the same SNR). This is because higher-order modulation schemes are more sensitive to noise and channel effects, requiring a significantly higher SNR to achieve an acceptable BER.

5. Conclusion

This paper investigates the optimal number of bits for a fractional-bit OFDM system, employing a 4096-point FFT and IFFT. Through extensive MATLAB simulations, our findings reveal that a 13-bit precision is the most effective for processing, offering the best performance. We also observed that a range of 10 to 13 bits provides consistently good results.

Furthermore, this study underscores the critical role of accurate channel estimation and compensation. The simulation results demonstrate a significant rise in the BER when additional channels are introduced. Even after applying signal estimation and compensation techniques, the BER remains notably high, particularly when compared to the impact of adding more AWGN channels at the same SNR. This suggests that in this specific scenario, channel impairments have a more pronounced negative effect on system performance than AWGN.

A crucial next step would be to validate these results through experimental implementation, for instance, on a FPGA. We should investigate how different channel estimation techniques, such as pilot-based or decision-directed methods, perform when using fixed-point arithmetic. The current study suggests channel effects are a major limiting factor, so exploring new estimation and compensation algorithms is vital to improving system robustness. This finding is crucial for hardware implementation, as it directly impacts the complexity, power consumption, and cost of the OFDM receiver. Highlighting this practical contribution would add significant value to the paper.

References

[1] T. Tyagi, R. P. Giddings and J. M. Tang, "Real-Time Demonstration of Concurrent Upstream and Inter-ONU Communications in Hybrid OFDM DFMA PONs," in *IEEE Photonics Technology Letters*, vol. 35, no. 3, 1 Feb.1, 2023, pp. 148-151.

[2] A. Farzamnia, N. W. Hlaing, M. K. Haldar and J. Rahebi, "Channel estimation for sparse channel OFDM systems using least square and minimum mean square error techniques," *2017 International*

Conference on Engineering and Technology (ICET), Antalya, Turkey, 2017, pp. 1-5.

[3] J. Armstrong, "OFDM for Optical Communications," in *Journal of Lightwave Technology*, vol. 27, no. 3, pp. 189-204, Feb.1, 2009,

[4] B. Bunsri, K. Puntsri and A. Yindeemak, "FPGA Implementation of IFFT-2048 Points for High Speed Transmitter OFDM Communication Systems with 64-QAM Mapping," *2024 12th International Electrical Engineering Congress (iEECON)*, Pattaya, Thailand, 2024, pp. 1-4.

[5] J. Jankovic, M. Subotic and V. Marinkovic, "One solution of the accurate summation using fixed-point accumulator," *2015 23rd Telecommunications Forum Telfor (TELFOR)*, Belgrade, Serbia, 2015, pp. 508-511

[6] K. Puntsri and W. Partipralam, "System Performance Investigation of OFDM Channel Qualization for 5G using Least Squares with Various Interpolation Methods," *2023 20th International Conference on Electrical Engineering/Electronics, Computer, Telecommunications and Information Technology (ECTI-CON)*, Nakhon Phanom, Thailand, 2023, pp. 1-4.